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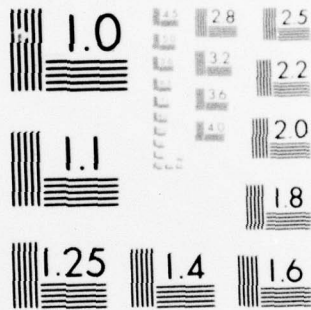
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EQUIVALENT GAIN FORMS
FOR CTD FILTERS,

By

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Stanley A. White

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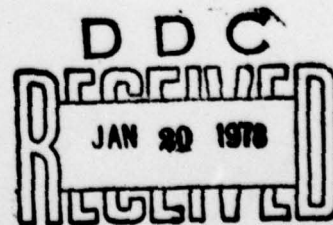
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EQUIVALENT GAIN FORMS FOR CTD FILTERS

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Abstract

There are three equivalent forms for expressing the gains of transversal analog filters which are mechanized with charge-transfer devices (CTD's). Due to charge-transfer inefficiency (CTI), η , the equivalence of expression is not obvious. This paper presents a simple tabulation of equivalences, their uses, and their derivations. This problem and solution also extend to include the recursive case. Arbitrary discrete-time recursive transfer functions are often mechanized digitally as second-order sections. Equivalent analog realization using charge-transfer devices cannot use the same gain parameters because of the effects (again) of charge-transfer inefficiency. This paper further includes an expression for corrected gains to compensate for this charge-transfer inefficiency in the recursive structure.

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Introduction

A wealth of design material exists to guide the designer of discrete-time transversal filters.¹ These design aids determine the tap weights, or gains, for filters of the form.

$$g(z) = \sum_{k=0}^K a_k z^{-k} \quad (1)$$

where the gain values are given by the a_k 's and where z^{-1} is the unit delay operator. Digital implementation of such filters is a straightforward procedure,² but analog implementation is more difficult.³ A digital delay of a single sample time, z^{-1} is realized precisely. An analog delay which uses charge-transfer devices (CTD) does not exhibit the ideal transfer function of z^{-1} , but instead the transfer function is

$$D_1(z) = \frac{z^{-1}(1-\eta)}{1-\eta z^{-1}}$$

where η is the charge-transfer inefficiency.⁴ If one were to attempt to mechanize the transfer function (1) by CTD's directly using the unmodified tap weights, he would obtain a new transfer function:

$$f(z) = \sum_{k=0}^K a_k \left(\frac{1-\eta}{1-\eta z^{-1}} \right)^k z^{-k} \quad (2)$$

which provides the desired transfer function only when $\eta=0$. Now, notice that a_k is the gain which is set by actual multipliers, but the effect of the η on the transfer function can be considerable and need be accounted for.

We wish to express (2) in a form similar to (1) because of the ready accessibility of design aids, however, (2) has a denominator of order K

while (1) has none. We have two options: eliminate the denominator in (2) by using a power-series expansion and truncating after the K th term, or acknowledge that the denominator is there. We choose the latter procedure which leads to:

$$f(z) = \frac{\sum_{j=0}^K b_j z^j}{(z-\eta)^K} = \frac{g(z)}{(z-\eta)^K} \quad (3a)$$

or, equivalently,

$$f(z) = \frac{\sum_{j=0}^K c_j z^{-j}}{(1-\eta z^{-1})^K} = \frac{z^{-K} g(z)}{(1-\eta z^{-1})^K} \quad (3b)$$

We are going to design a CTD filter by first determining the b 's (or the c 's) then the actual multiplier gains (the a 's). These b 's and c 's are obviously not the actual multiplier gains; however, these expressions of (3) and (4) are much closer to the "standard" form of (1) because their numerators are exactly in the form of (1).

The design procedure is now executed in 3 steps:

1. The desired transfer function, $\hat{f}(z)$, is determined. We then modify it to form \hat{g} in the step below.
2. By establishing FIR filter design procedures, such as described in references 1 and 2, the gains b_j or c_j are obtained from

$$\hat{g}(z) = \hat{f}(z)(z-\eta)^K = \sum_{j=0}^K b_j z^j = g(z) \quad (4a)$$

or

$$z^{-K} g(z) = f(z)(1-nz^{-1})^K = \sum_{j=0}^K c_j z^{-j} \quad (4b)$$

3. The actual multiplier gains a_k , are determined from the b_j or c_j :

$$a_k = \frac{\sum_{j=K-k}^K \binom{j}{k} b_j n^{j-K+k}}{(1-n)^k} \quad (5a)$$

$$= \frac{\sum_{j=K-k}^K \binom{j}{k} c_{N-j} n^{j-K+k}}{(1-n)^k} \quad (5b)$$

which is derived in Section II.

Above we have a synthesis procedure. Now let's examine the solution to the inverse problem, the analysis procedure; i.e., given the values of a_k , determine the filter impulse and frequency responses. Existing software abounds to develop the answer from the b 's or c 's.

The analysis procedure is executed in 4 steps:

1. The filter structure is given in the form of (1).
2. The coefficients b_j or c_j are obtained from the a_k by

$$b_j = \sum_{k=j}^K a_{K-k} (1-n)^{K-k} \binom{k}{j} (-n)^{k-j} \quad (6a)$$

$$= c_{K-j}$$

and

$$c_j = \sum_{k=K-j}^K a_{K-k} (1-n)^{K-k} \binom{k}{K-j} (-n)^{k-K+j} \quad (6b)$$

$$= b_{K-j}$$

3. FIR filter analysis programs can be used to evaluate the performance of:

$$g(z) = \sum_{j=0}^K b_j z^j \quad (7a)$$

or

$$z^{-K} g(z) = \sum_{j=0}^K c_j z^j \quad (7b)$$

4. Performance of the actual filter can be evaluated from

$$f(z) = g(z)(z-n)^{-K} = z^{-K} g(z)(1-nz^{-1})^{-K} \quad (7c)$$

II. Derivations

Since each individual linearized CTD cell can be represented by the transfer function

$$D_1(z) = \frac{1-n}{1-nz^{-1}} z^{-1} = \frac{1-n}{z-n} \quad (8)$$

the N^{th} order tapped delay line can be described by

$$f(z) = a_0 + a_1 \frac{1-n}{z-n} + a_2 \left(\frac{1-n}{z-n} \right)^2 + \dots + a_K \left(\frac{1-n}{z-n} \right)^K \quad (9a)$$

$$= \sum_{k=0}^K a_k \left(\frac{1-n}{z-n} \right)^k \quad (9b)$$

$$= \frac{\sum_{k=0}^K a_k (1-n)^k (z-n)^{K-k}}{(z-n)^K} = \frac{g(z)}{(z-n)^K} \quad (9c)$$

The numerator of (9c) can be rewritten as

$$g(z) = \sum_{k=0}^K a_k (1-n)^k (z-n)^{K-k} \quad (10)$$

$$= \sum_{k=0}^K a_{N-k} (1-n)^{K-k} (z-n)^k$$

by redefining k as $K-k$. Next we'll expand $(z-n)^k$, so that now

$$g(z) = \sum_{k=0}^K a_{K-k} (1-n)^{K-k} \sum_{r=0}^k \binom{k}{r} z^{k-r} (-n)^r \quad (11)$$

$$= \sum_{j=0}^K z^j \sum_{k=j}^K a_k \binom{k}{j} (-n)^{k-j}$$

where $j=k-r$, and

$$a_k = a_{K-k} (1-n)^{K-k} \quad (12a)$$

and

$$b_{jk} = \sum_{k=j}^K (-n)^{k-j} \binom{k}{j} (-n)^{k-j} \quad (12b)$$

Equations (9c), (11), (12a) and (12b) combine to yield (3a) with b_j defined as in (6a). When (3a) is divided by z^K , (3b) and (6b) emerge. Equations (10) and (7a) can be combined to give

$$g(z) = \sum_{j=0}^K a_{K-j} (1-n)^{K-j} (z-n)^j = \sum_{j=0}^K b_j z^j \quad (13)$$

then

$$\lim_{z \rightarrow \eta} \frac{\partial^k g(z)}{\partial z^k} = a_{K-k} (1-\eta)^{K-k} k! \quad (13)$$

$$= \sum_{j=k}^K \frac{j!}{(j-k)!} b_j \eta^{j-k}$$

yielding

$$\begin{aligned} a_{N-k} &= \sum_{j=k}^K \frac{j!}{k!(j-k)!} \frac{b_j \eta^{j-k}}{(1-\eta)^{K-k}} \\ &= \frac{\sum_{j=k}^K \binom{j}{k} b_j \eta^{j-k}}{(1-\eta)^{K-k}} \end{aligned} \quad (14)$$

which can be expressed in the form of (5a).

From (3a) and (3b) one can see that $b_j = c_{K-j}$ which when substituted in (5a) give (5b).

III. The Recursive Case

Here we shall introduce one additional level of complexity. For reasons of economics we often find CTD transversal filters of large order; CTD's come that way. Since stability problems mitigate against large-order recursive filters, the order is restricted to a small number, like two. In order to capitalize on the large number of CTD's which one can economically make, we find structures like

$$F(z^N) = \frac{1}{1 + Az^{-N} + Bz^{-2N}}$$

this can be used as a second-order filter multiplexed across N data streams, but cursed with crosstalk. This can also be used as a single channel filter with a periodic frequency response. In either case, the following argument holds.

Suppose that the transfer function of an ideal discrete-time processor is

$$G(z) = \frac{p(z)}{q(z)} \quad (15)$$

its poles and zeros are given by the respective solutions to

$$q(z) = 0 \quad \text{and} \quad p(z) = 0.$$

The singularities in each case are given by expressions of the form

$$z = z_k. \quad (16)$$

If the processor is to be time-shared between N channels, we could rewrite (6) as

$$z = z_1^N = z_k \quad (17)$$

Using the CTD mechanization

$$\left[\frac{z_1 - \eta}{1 - \eta} \right]^N = z_k \quad (18)$$

and we see that the filter singularities have been modified to

$$z_1^N = \left[(1-\eta)z_k^{1/N} + \eta \right]^N \quad (19)$$

We would like to prewarp z_k to z'_k so that (17) is preserved in (19), i.e., so that

$$z_1^N = z_k = \left[(1-\eta)z'_k^{1/N} + \eta \right]^N \quad (20)$$

Solving the above for z'_k yields the prewarped singularity location

$$z'_k = \left[\frac{z_k^{1/N} - \eta}{1 - \eta} \right]^N \quad (21)$$

Now let's relate this to filter gain values. The conventional description of singularities is usually given in polar coordinates so that

$$z_k = r_k \angle \theta_k \quad (22)$$

Conjugate-complex pairs of singularities, z_k and z_k^* , appear in the characteristic equation:

$$0 = (z - z_k)(z - z_k^*) = z^2 + A_k z + B_k \quad (23)$$

where

$$A_k = 2r_k \cos \theta_k = -2 \operatorname{Re}[z_k] \quad (24)$$

and

$$B_k = r_k^2 = |z_k|^2 \quad (25)$$

and where A_k and B_k are filter gain values.

The results of the prewarping will be incorporated into the gain values:

$$A'_k = 2 \operatorname{Re}[z'_k] = -2 \sqrt{B'_k} \cos \quad (26)$$

$$\left(N \tan^{-1} \frac{r_k^{1/N} \sin \frac{\theta_k}{N}}{r_k^{1/N} \cos \frac{\theta_k}{N} - \eta} \right)$$

and

$$B'_k = \left[\frac{r_k^{2/N} - 2\eta r_k^{1/N} \cos \frac{\theta_k}{N} + \eta^2}{(1-\eta)^2} \right]^N \quad (27)$$

which indeed places the singularities of the CTD-mechanized filter in their proper locations. For large N we may simplify (26) and (27) by the approximations:

$$A'_k \approx -2\sqrt{B'_k} \cos \frac{\theta_k}{1-\eta r_k^{-1/N}} \quad (28)$$

and

$$B'_k \approx B_k \left[\frac{1-\eta r_k^{-1/N}}{1-\eta} \right]^{2N} \quad (29)$$

The gain for real singularity is given directly by (21).

IV. References

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